MAT 67 **Sample Homework**
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Problem: Let $z \in \mathbb{C}$. Prove that $Im(z) = 0$ if and only if $Re(z) = z$.

Solution: Let us write $z = x + iy$. Now suppose $Im(z) = 0$. We want to show that $Re(z) = z$.

Since $Im(z) = 0$, we have $y = 0$. Therefore $z = x + i.0 = x$. But we know that $Re(z) = x$. Therefore $z = x = Re(z)$.

Conversely, suppose $Re(z) = z$. Then we have $x = x + iy$. Which implies that $iy = 0$ i.e., $y = 0$ (since $i \neq 0$).

Possible wrong solutions

Solution 1: Let us write $z = x + iy$. Now suppose $Im(z) = 0$. We want to show that $Re(z) = z$.

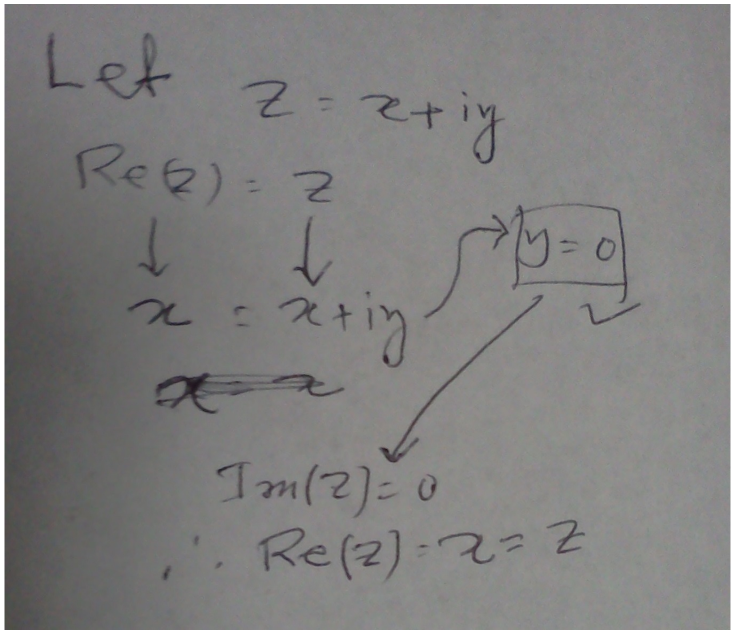
Since $Im(z) = 0$, we have $y = 0$. Therefore $z = x + i.0 = x$. But we know that $Re(z) = x$. Therefore $z = x = Re(z)$.

Remark: (i)Clearly, the above is an incomplete solution. It is missing the second part ("only if" part).
(ii) This is a common proof writing mistake. Read carefully if there is any "if and only if" phrase. If so, then you are required to prove it from both sides. See the following example.
(iii) True or False? "An integer is multiple of 6 if and only if it is divisible by 2".
Answer: Let us try to prove the statement and see where we get stuck. Suppose an integer is multiple of 6, say $6n$. Then clearly it is divisible by 2.
Conversely, suppose an integer is divisible by 2. Is it necessarily true that it is multiple of 6? NO, for example consider the integer 4. Which is divisible by 2 but not a multiple of 6. Therefore the statement is FALSE.

(iv) If you didn't think about the converse part, you would probably say that the statement is TRUE!
(v) See Wikipedia to know more about 'if and only if' or equivalently 'iff'.
(vi) (Do it by yourself) True or False? "An integer is multiple of 6 only if it is divisible by 2"

Solution 2: $Im(z) = 0$. $Re(z) = x + i.0 = x = z$. Hence done.

OR



Let $z = x+iy$

$Re(z) = z$

$\downarrow \qquad \downarrow$

$x = x+iy \rightarrow \boxed{y=0}$ ✓

$\cancel{x = x}$

$Im(z) = 0$

$\therefore Re(z) = x = z$

Remark:(i)I didn't get it! Write your solutions in complete English sentences.
(ii) Any such solution will receive a <u>ZERO</u>.