MAT 67 Max time 20mins **Quiz 4**

Name:____ August 28, 2014

All unnecessary electronics must be turned off and out of sight. This means no cellular phones, iPods, wearing of headphones, or anything of the sort. This is a closed book and closed notes test.

Problem 1: Consider the following matrix

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}.$$

(i) *(5 points)* Compute the characteristic polynomial $P(\lambda)$. You do not need to simplify your answer.

(ii) *(5 points)* Find the eigenvalues of A.

(iii) *(3 points)* Is the matrix A invertible? Explain your answer.

Solution:

(i) The characteristic polynomial $P(\lambda)$ is given by

$$\begin{aligned} P(\lambda) &= det(A - \lambda I) \\ &= \begin{vmatrix} -1-\lambda & 0 & 1 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & 4-\lambda \end{vmatrix} \\ &= (-1-\lambda)\begin{vmatrix} 3-\lambda & 2 \\ 0 & 4-\lambda \end{vmatrix} \quad \text{(expanding w.r.to the first column)} \\ &= (-1-\lambda)(3-\lambda)(4-\lambda). \end{aligned}$$

(ii) Eigenvalues of A are the roots of the characteristic polynomial. Solving $(-1-\lambda)(3-\lambda)(4-\lambda) = 0$, we obtain the eigenvalues $\lambda = -1, 3, 4$.

(iii) Notice that $det(A) = -1 \times 3 \times 4 = -12$. Since $det(A) \neq 0$, the matrix A is invertible.

Problem 2: *(7 points)* Find all possible inversion pairs in the following permutation

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 2 & 4 & 6 \end{pmatrix}.$$

Solution: Inversion pairs which look like $(1, j)$ are $(1, 2)$ and $(1, 4)$. There is no inversion pair of the form $(2, j)$. Inversion pairs of the form $(3, j)$ are $(3, 4), (3, 5)$. Also there are no inversion pairs of the forms $(4, j), (5, j)$, or $(6, j)$. Therefore all possible inversion pairs are $(1, 2), (1, 4), (3, 4)$, and $(3, 5)$.