MAT 67 Max time 20mins **Quiz 3**

Name:____August 20, 2014

All unnecessary electronics must be turned off and out of sight. This means no cellular phones, iPods, wearing of headphones, or anything of the sort. This is a closed book and closed notes test.

Problem 1: *(8 points)* Define the map $T : \mathbb{R}^2 \to \mathbb{R}^2$ as $T(x, y) = (2x + 3y, x - y)$. Find the matrix of T with respect to the canonical basis $\{(1, 0), (0, 1)\}$.

Solution: We compute

$$\begin{aligned} T((1,0)) &= (2,1) = 2(1,0) + 1(0,1) \\ T((0,1)) &= (3,-1) = 3(1,0) + (-1)(0,1). \end{aligned}$$

Therefore the matrix of T with respect to the canonical basis is

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}.$$

Problem 2: *(12 points)* Show that the following subspace U of $\mathbb{R}^5$ can not be the null space of any linear map $T : \mathbb{R}^5 \to \mathbb{R}^2$.

$$U = \{(x_1, x_2, x_3, x_4, x_5) | x_1 = 2x_2, x_3 = x_4 = x_5\} \subset \mathbb{R}^5.$$

This is a mini proof-writing problem. So write your solution using complete English sentences. *[Hint: Find the dimension of the above subspace, and then use the dimension formula].*

Solution: First of all, we notice that $dim(U) = 2$. Because $\mathbb{R}^5$ has dimension 5, and U is obtained from $\mathbb{R}^5$ by imposing three independent constraints namely, $x_1 = 2x_2, x_3 = x_4, x_4 = x_5$.

Now suppose there is a linear map $T : \mathbb{R}^5 \to \mathbb{R}^2$ such that

$$null(T) = \{(x_1, x_2, x_3, x_4, x_5) | x_1 = 2x_2, x_3 = x_4 = x_5\}.$$

Then using the dimension formula we have

$$\begin{aligned} dim(\mathbb{R}^5) &= dim(null(T)) + dim(range(T)) \\ i.e., \ dim(range(T)) &= 5 - 2 = 3. \end{aligned}$$

But this is a contradiction. Because $range(T) \subset \mathbb{R}^2$, therefore $dim(range(T)) \leq 2$.