MAT 67 Max time 20mins **Quiz 2**

Name:____ August 14, 2014

All unnecessary electronics must be turned off and out of sight. This means no cellular phones, iPods, wearing of headphones, or anything of the sort. This is a closed book and closed notes test.

Problem 1: *(5 points)* Find the dimension of the following vector space

$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 | x_1 = x_2, x_2 = x_3, x_1 + x_3 = 2x_2\}.$$

Explain your answer.

Solution: We know that $\dim(\mathbb{R}^4) = 4$. In the above set V, two independent linear conditions are imposed on $\mathbb{R}^4$, namely $x_1 = x_2$ and $x_2 = x_3$ (note that the condition $x_1 + x_3 = 2x_2$ can be obtained from $x_1 = x_2$, $x_2 = x_3$ just by adding them). Therefore $\dim(V)=\dim(\mathbb{R}^4) - 2 = 4 - 2 = 2$.

Problem 2: *(5 points)* Consider the following set of 2×2 matrices,

$$S = \left\{ \begin{bmatrix} a & a+2b+1 \\ a+2b & a \end{bmatrix} \mid a, b \in \mathbb{R} \right\}.$$

Is the set S a vector space under the usual matrix addition and scalar multiplication? Explain your answer.

Solution: NO. Because the above set is missing the zero vector i.e., $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin S$.

Suppose, if possible $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$. Then there exists $a, b \in \mathbb{R}$ such that $\begin{bmatrix} a & a+2b+1 \\ a+2b & a \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Then we must have $a = 0, a + 2b = 0, a + 2b + 1 = 0$. The first and second equation imply that $a = 0, b = 0$. Then the third equation implies that $1 = 0$!, Contradiction.

Problem 3: *(5 points)* Is it true that $\text{span}\{(1,1,0,0),(0,1,1,0),(0,0,1,1)\} = \mathbb{R}^4$? Explain your answer.

Solution: No, because we need at least four vectors to span the $\mathbb{R}^4$.

Problem 4: *(5 points)* True or False? *"Let $v_1, v_2, v_3, v_4 \in \mathbb{R}^4$ such that $span\{v_1, v_2, v_3, v_4\} = \mathbb{R}^4$, then v_1, v_2, v_3, v_4 are linearly independent".* Explain your answer.

Solution: Yes. Because if v_1, v_2, v_3, v_4 are linearly dependent then we can get rid of one of them, say v_4, and still they will span the same space i.e., $span\{v_1, v_2, v_3\} = span\{v_1, v_2, v_3, v_4\} = \mathbb{R}^4$. But three vectors can not span the $\mathbb{R}^4$, contradiction! Therefore v_1, v_2, v_3, v_4 must be linearly independent.