MAT 67 Max time 20mins **Quiz 1**

Name:____ August 7, 2014

All unnecessary electronics must be turned off and out of sight. This means no cellular phones, iPods, wearing of headphones, or anything of the sort. This is a closed book and closed notes test.

Problem 1: Find the multiplicative inverse (i.e., z^{-1}) of the complex number $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$.

Solution: The multiplicative inverse of z is given by

$$\begin{aligned} z^{-1} &= \frac{1}{\frac{1}{2} + i\frac{\sqrt{3}}{2}} \\ &= \frac{\frac{1}{2} - i\frac{\sqrt{3}}{2}}{\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)} \\ &= \frac{\frac{1}{2} - i\frac{\sqrt{3}}{2}}{\frac{1}{4} + \frac{3}{4}} \\ &= \frac{1}{2} - i\frac{\sqrt{3}}{2} \end{aligned}$$

Problem 2: Solve the equation $z^{10} + 1024 = 0$.

Solution: We rewrite the above equation as

$$z^{10} = -1024 = 1024e^{i\pi}.$$

Solving the above equation we obtain

$$z = (1024)^{1/10}e^{i\frac{\pi}{10} + i\frac{2k\pi}{10}} = 2e^{i\frac{\pi}{10} + i\frac{2k\pi}{10}},$$

where $k = 0, 1, \ldots, 9$.

Problem 3: Let a be a real number. Construct a polynomial $p : \mathbb{C} \to \mathbb{C}$ of degree at most two such that $p(-1) = a, p(-2) = a^2$, and $p(-3) = a^3$.

Solution: The quadratic polynomial that satisfies the above conditions is

$$\begin{aligned} p(z) &= a\frac{(z+2)(z+3)}{(-1+2)(-1+3)} + a^2\frac{(z+1)(z+3)}{(-2+1)(-2+3)} + a^3\frac{(z+1)(z+2)}{(-3+1)(-3+2)} \\ &= \frac{a}{2}(z+2)(z+3) - a^2(z+1)(z+3) + \frac{a^3}{2}(z+1)(z+2). \end{aligned}$$