MAT 21D Max time 20mins **Quiz 2**

Name:____ August 9, 2016

All unnecessary electronics must be turned off and out of sight. This means no calculator, cellular phones, iPods, wearing of headphones, or anything of the sort. This is a closed book and closed notes test.

Problem 1: *(10 points)* Integrate $f(x,y) = \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}$ over the region $1 \le x^2 + y^2 \le e$.

[Hint: Change it into a polar integral]

Solution: step 1: $x = r\cos\theta,\ y = r\sin\theta$ and $dydx = rdrd\theta$.

step 2: Finding the limits of r; From the picture (which you should have drawn by yourself), it is clear that for a given theta, r varies from 1 to $\sqrt{e}$. *(2+1 points)*

step 3: From the same picture we obtain the range of θ is from 0 to 2π. *(1 point)*

Therefore the polar form of the given integral is

$$\begin{aligned}
\int_0^{2\pi}\int_1^{\sqrt{e}} \frac{\ln r^2}{r}\, rdrd\theta &= 2\int_0^{2\pi}\int_1^{\sqrt{e}} \ln r\ drd\theta \qquad \text{(2 points)} \\
&= 2\int_0^{2\pi} \left[r\ln r - r|_1^{\sqrt{e}} \right]\ d\theta \qquad \text{(2 points)} \\
&= 2(-\sqrt{e}/2 + 1)\int_0^{2\pi} d\theta \\
&= 2\pi(2-\sqrt{e}) \qquad \text{(2 points)}.
\end{aligned}$$

Problem 2: *(10 points)* Find the volume of the region bounded above by the paraboloid $z = 5 - x^2 - y^2$ and below by the paraboloid $z = 4x^2 + 4y^2$.

Solution: These two paraboloid intersects along the curve $5 - x^2 - y^2 = 4x^2 + 4y^2$ i.e., $x^2 + y^2 = 1$. *(2 points)*

Using the picture (which you should have drawn by yourself *(2 points)*, somewhat similar to the Figure 15.31) and cylindrical coordinate system, the volume of the solid is given by

$$\begin{aligned}
\textit{(3 points)} \quad \int_0^{2\pi}\int_0^1\int_{4r^2}^{5-r^2} dz\, r dr d\theta &= \int_0^{2\pi}\int_0^1 (5-5r^2) r\, dr d\theta \\
&= 5\int_0^{2\pi}\left[\frac{r^2}{2} - \frac{r^4}{4}\bigg|_0^1\right] d\theta \\
&= 5 \times \frac{1}{4} \times 2\pi = \frac{5\pi}{2}. \qquad \textit{(3 points)}
\end{aligned}$$