MAT 21D Max time 20mins **Quiz 1**

Name:____ August 4, 2016

All unnecessary electronics must be turned off and out of sight. This means no cellular phones, iPods, wearing of headphones, or anything of the sort. This is a closed book and closed notes test.

Problem 1: *(10 points)* Sketch the region of integration, reverse the order of integration and evaluate the integral.

$$\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy\,dx}{y^4+1}.$$

Solution: Draw the region by yourself (2 points).

$$\begin{aligned}
\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy\,dx}{y^4+1} &= \int_0^2 \int_0^{y^3} \frac{1}{y^4+1}\,dxdy \qquad \text{(5 points)} \\
&= \int_0^2 \frac{y^3}{1+y^4}\,dy \\
&= \frac{1}{4}\ln(1+y^4)\Big|_0^2 \qquad \text{(2 points)} \\
&= \frac{1}{4}\ln 17 \qquad \text{(1 point)}.
\end{aligned}$$

Problem 2: *(10 points)* Find the volume of the solid whose top surface is given by the equation $f(x,y) = 4 - x^2 - y$ and the base is bounded by $x = 0$, $y = 0$, $4 - x^2 - y = 0$ on the xy-plane.

Solution: *Clarification: There is an ambiguity in the statement of the problem. The base region can be either in the first or in the second quadrant. You may consider either of those two.*

Base of the solid on the xy-plane is bounded by the x axis, y axis and the curve $0 = 4 - x^2 - y$. Therefore the volume of the solid is given by

$$\begin{aligned}
\int_0^2 \int_0^{4-x^2} (4 - x^2 - y)\, dydx &= \int_0^2 \left[4y - x^2 y - y^2/2 \Big|_{y=0}^{4-x^2} \right] dx \qquad \text{(5 points)} \\
&= \frac{1}{2} \int_0^2 (4 - x^2)^2\, dx \\
&= \frac{1}{2} \int_0^2 (16 - 8x^2 + x^4)\, dx \\
&= \frac{1}{2} \left(16x - \frac{8}{3}x^3 + \frac{x^5}{5}\right)\Big|_0^2 \qquad \text{(2 points)} \\
&= \frac{1}{2}(32 - 64/3 + 32/5) = \frac{128}{15} \qquad \text{(1 point).}
\end{aligned}$$

Sketch the region (2 points).