MAT 21D Max time: 1 hr 20 mins **Practice Midterm 2**

Name:____ August 24, 2016

Before the exam begins:

- Write your name above.
- Turn off all electronics and keep them out of sight: no cellular phones, iPods, wearing of headphones, not even to tell time (and not even if it's just in airplane mode).
- You may bring hand written notes ONLY ON ONE SIDE of a half page (where full page = max A4).

As soon as the exam starts:

- Take a quick breath to relax! If you have truly worked through all the homework problems then you will do fine!
- Check that you have all **seven** pages of the exam. (The number of pages includes this cover page.)

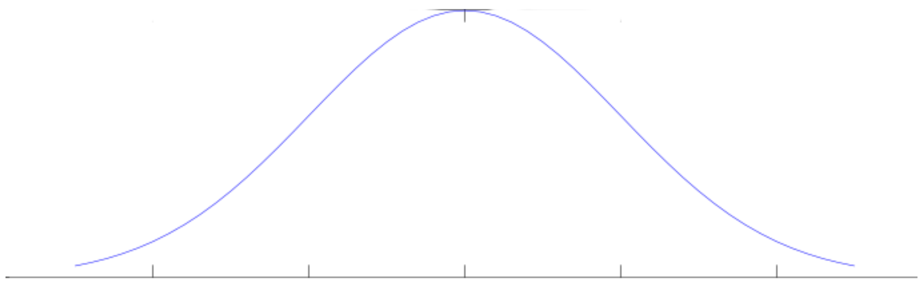
During the exam:

- Keep your eyes on your own exam!

Note that the exam length is exactly 1 hr 20 mins. When you are told to stop, you must stop **IMMEDIATELY**. This is in fairness to all students. Do not think that you are the exception to this rule.

Problem	1	2	3	4	5	6	**Total**
Score							

Problem 1: *(10 points)* Find the area of the region bounded between the graph of the function $f(x) = e^{-x^2}$ and the x axis.



Solution:

Problem 2: *(10 points)* Draw the region of the integration and evaluate the following integral

$$\int_0^1 \int_{2y}^2 4\cos(x^2)\, dx dy$$

Solution:

Problem 3:*(15 points)* Find the mass of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where the density is given by $\delta(x, y, z) = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}}$. *[Hint: Let x=au, y=bv, and z=cw. Then find the mass of an appropriate region in uvw-space.]*

Solution:

Problem 4: *(20 points)* Consider the following space curve

$$\vec{r}(t) = (e^t \cos t)\hat{i} + (e^t \sin t)\hat{j} + 2\hat{k}.$$

(a) *(5 points)* Find the length of the curve from the point $(1, 0, 2)$ to the point $(0, e^{\pi/2}, 2)$.
(b) *(15 points)* Find the tangent vector $\vec{T}$, unit normal vector $\vec{N}$ and the curvature κ.

Solution:

Problem 5:*(10 points)* Consider the vector field $\vec{F} = 2x\hat{\imath} - 3y\hat{\jmath}$, and the circle $\vec{r}(t) = (a\cos t)\hat{\imath} + (a\sin t)\hat{\jmath}$, $0 \leq t \leq 2\pi$. Find the circulation of the field along the circle, and the flux of the field across the circle.

Solution:

Problem 6: *(15 points)* Consider a thick spherical shell whose inner radius is a and outer radius is b and the density is $\delta = 1$. Find the moment of inertia of this spherical with respect to a diameter.

Solution: