MAT 21D Max time: 1 hr 20 mins ~~**Midterm**~~

Name:____ August 18, 2016

Before the exam begins:

- Write your name above.
- Turn off all electronics and keep them out of sight: no cellular phones, iPods, wearing of headphones, not even to tell time (and not even if it's just in airplane mode).
- You may bring hand written notes ONLY ON ONE SIDE of a half page (where full page = max A4).

As soon as the exam starts:

- Take a quick breath to relax! If you have truly worked through all the homework problems then you will do fine!
- Check that you have all **seven** pages of the exam. (The number of pages includes this cover page.)

During the exam:

- Keep your eyes on your own exam!

Note that the exam length is exactly 1 hr 20 mins. When you are told to stop, you must stop **IMMEDIATELY**. This is in fairness to all students. Do not think that you are the exception to this rule.

Problem	1	2	3	4	5	6	**Total**
Score							

Problem 1: *(10 points)* Evaluate the following integral

$$\int_0^\infty e^{-x^2}\ dx.$$

[Hint: Consider the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\ dydx$*.]*

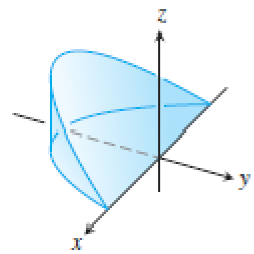
Solution:

Problem 2: *(10 points)* Sketch the region of integration, and then evaluate the following integral

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} \; dydx$$

Solution:

Problem 3: *(15 points)* Find the *y-coordinate* of the centroid of the wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$, where the density is given by $\delta(x, y, z) = 1$.



Solution:

Problem 4: *(15 points)* Consider the following space curve

$$\vec{r}(t) = (e^t \cos t)\hat{\imath} + (e^t \sin t)\hat{\jmath} + 2\hat{k}.$$

(a) Find the length of the curve from the point $(1, 0, 2)$ to the point $(0, e^{\pi/2}, 2)$.
(b) Find the tangent vector $\vec{T}$, unit normal vector $\vec{N}$ and the curvature κ.

Solution:

Problem 5: *(15 points)* Find the center of mass of a thin wire lying along the curve $\vec{r}(t) = t\hat{i} + 2t\hat{j} + (2/3)t^{3/2}\hat{k}$, $0 \le t \le 2$, if the density is $\delta = 3\sqrt{5+t}$.

Solution:

Problem 6: *(15 points)* Consider the vector field $\vec{F} = (y\sin z)\hat{i} + (x\sin z)\hat{j} + (xy\cos z)\hat{k}$.
(a) Is it a conservative vector field?
(b) Find the work done while moving a particle from $(0,0,0)$ to $(1,1,1)$ in this vector field.

Solution: