MAT 21D Max time: 1 hr 20 mins **Practice Final 2**

Name:____ September 4, 2016

Before the exam begins:

- Write your name above.
- Turn off all electronics and keep them out of sight: no calculator, no cellular phones, iPods, wearing of headphones, not even to tell time (and not even if it's just in airplane mode).
- You may bring hand written notes ONLY ON ONE SIDE of a half page (where full page = max A4).

As soon as the exam starts:

- Take a quick breath to relax! If you have truly worked through all the homework problems then you will do fine!
- Check that you have all **eight** pages of the exam. (The number of pages includes this cover page.)
- There is an *extra credit problem* on the last page.

During the exam:

- Keep your eyes on your own exam!

Note that the exam length is exactly 1 hr 20 mins. When you are told to stop, you must stop **IMMEDIATELY**. This is in fairness to all students. Do not think that you are the exception to this rule.

Problem	1	2	3	4	5	6	**Total**
Score							

Problem 1: *(10 points)* Find the mass of the solid that is bounded above by the cylinder $z = 4 - x^2$, on the sides by the cylinder $x^2 + y^2 = 4$, and below by he xy-plane. The density of the solid is given by $\delta(x, y, z) = \sqrt{x^2 + y^2}$.

Solution:

Problem 2: *(10 points)* Find the moment of inertia with respect to a diameter of the solid sphere of radius a.

Solution:

Problem 3: *(15 points)* Consider the space curve

$$\vec{r}(t) = (\cos^3 t)\hat{\imath} + (\sin^3 t)\hat{\jmath}, \quad 0 \le t \le \pi/2.$$

(a) *(5 points)* Find the length of the curve.
(b) *(5 points)* Find the curvature κ.

Solution:

Problem 4: *(15 points)* Consider the vector field

$$\vec{F} = (2x \ln y - yz)\hat{\imath} + \left(\frac{x^2}{y} - xz\right)\hat{\jmath} - xy\hat{k}.$$

Find the work done while moving a particle in the above vector field from $(1, 2, 1)$ to $(2, 1, 1)$ along the straight line.

Solution:

Problem 5: *(15 points)* Let S be the "cup" surface formed by rotating the curve $x = \sin z, y = 0, 0 \leq z \leq \pi/2$ around the $z-$axis. Find the mass of the surface, where the density is given by $\delta(x, y, z) = \sqrt{1 - x^2 - y^2}$.

Solution:

Problem 6: *(15 points)* Let $\hat{n}$ be the outer unit normal (normal away from the origin) of the parabolic shell $S: \quad 4x^2 + y + z^2 = 4,\ y \geq 0$, and let

$$\vec{F} = \left(-z + \frac{1}{2+x}\right)\hat{i} + (\tan^{-1} y)\hat{j} + \left(x + \frac{1}{4+z}\right)\hat{k}.$$

Find the value of

$$\int\int_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n}\, d\sigma.$$

Solution:

Extra Credit:*(2 points)* I'll think about it.