MAT 21D Max time: 1 hr 20 mins **Practice Final 1**

Name:____ September 4, 2016

Before the exam begins:

- Write your name above.
- Turn off all electronics and keep them out of sight: no calculator, no cellular phones, iPods, wearing of headphones, not even to tell time (and not even if it's just in airplane mode).
- You may bring hand written notes ONLY ON ONE SIDE of a half page (where full page = max A4).

As soon as the exam starts:

- Take a quick breath to relax! If you have truly worked through all the homework problems then you will do fine!
- Check that you have all **eight** pages of the exam. (The number of pages includes this cover page.)
- There is an <u>*extra credit problem*</u> on the last page.

During the exam:

- Keep your eyes on your own exam!

Note that the exam length is exactly 1 hr 20 mins. When you are told to stop, you must stop **IMMEDIATELY**. This is in fairness to all students. Do not think that you are the exception to this rule.

Problem	1	2	3	4	5	6	**Total**
Score							

Problem 1:*(10 points)* Evaluate the following integral

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx.$$

Solution:

Problem 2: *(10 points)* Find the mass of the solid spherical cap $x^2 + y^2 + z^2 \leq 25, z \geq 3$ *[which is obtained by cutting the solid sphere* $x^2 + y^2 + z^2 \leq 25$ *by the plane* $z = 3$*]*. The desity of the cap is given by $\delta = 1$.

Solution:

Problem 3: *(15 points)* Consider the following space curve

$$\vec{r}(t) = (e^t \cos t)\ \hat{i} + (e^t \sin t)\ \hat{j} + \sqrt{2}e^t\ \hat{k}.$$

(a) Find the curvature κ of above space curve at the point $t = 0$.

(b) Find the tangential component a_T and the normal component a_N of acceleration at $t = 0$.

Solution:

Problem 4: *(15 points)*

Consider the vector fields $\vec{F}_1$, $\vec{F}_2$, and the curve C given below

$$\vec{F}_1 = (3x^2y^2 + 2y)\ \hat{i} + (2x^3y - 3x)\ \hat{j},$$
$$\vec{F}_2 = (2y\cos x + 3x)\ \hat{i} + (y^2\sin x + 2y)\ \hat{j},$$
$$C:\ \vec{r}(t) = \cos^3 t\ \hat{i} + \sin^3 t\ \hat{j}, \quad 0 \le t \le 2\pi$$

(a) Find the area of the region enclosed by the curve C.

(b) Find the counter-clockwise circulation of the vector field $\vec{F}_1$ along the curve C.

(c) Find the outward flux of the vector field $\vec{F}_2$ across the curve C.

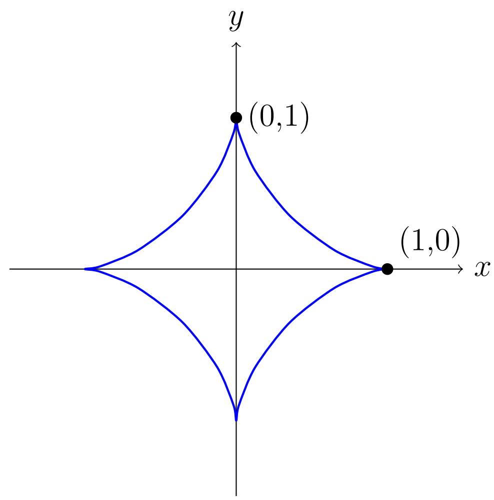


Figure 1: Graph of the curve C.

Solution:

Problem 5: *(15 points)* Find the moment of inertia of the thin spherical shell $x^2+y^2+z^2 = a^2$ with respect to any diameter of it. The density of the shell is $\delta = 1$.

Solution:

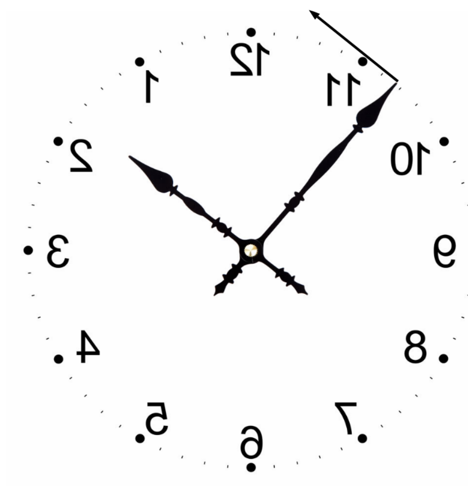
Problem 6:*(15 points)* Consider the ellipsoidal shell $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, and let

$$\vec{F} = \frac{y^2 x}{b^2}\hat{\imath} + \frac{z^2 y}{c^2}\hat{\jmath} + \frac{x^2 z}{a^2}\hat{k},$$

where a, b, c are constants. Find the outward flux of $\vec{F}$ across the surface of the ellipsoidal shell.

Solution:

Extra Credit: *(2 points)*



The arrow shows the direction of motion of the minute hand. Is the minute hand rotating clockwise or counter-clockwise?