Section 2.6

[5] $g(t) = \frac{1}{3}t^3 - 4t^2 + 2t$. Differentiating both sides with respect to t we have

$$\begin{aligned} g'(t) &= \frac{1}{3} \times 3t^2 - 4 \times 2t + 2 \\ &= t^2 - 8t + 2. \end{aligned}$$

Differentiating $g'(t)$ once again with respect to t we have

$$g''(t) = 2t - 8.$$

[12] $g(t) = -\frac{4}{(t+2)^2}$. Rewrite the function as $g(t) = -4(t+2)^{-2}$. Differentiating both sides with respect to t we have

$$\begin{aligned} g'(t) &= -4 \times (-2)(t+2)^{-3} \frac{d}{dt}[t+2] \\ &= 8(t+2)^{-3}. \end{aligned}$$

Differentiating once again with respect to t we have

$$\begin{aligned} g''(t) &= 8 \times (-3)(t+2)^{-4} \frac{d}{dt}[t+2] \\ &= -24(t+2)^{-4} \\ &= -\frac{24}{(t+2)^4}. \end{aligned}$$

[24] $f(t) = \sqrt{2t+3}$. Differentiating with respect to t we have

$$\begin{aligned} f'(t) &= \frac{d}{dt}\left[(2t+3)^{\frac{1}{2}}\right] \\ &= \frac{1}{2}(2t+3)^{\frac{1}{2}-1} \frac{d}{dt}[2t+3] \\ &= \frac{1}{2}(2t+3)^{-\frac{1}{2}} \cdot 2 \\ &= (2t+3)^{-\frac{1}{2}}. \end{aligned}$$

Similarly,

$$\begin{aligned} f''(t) &= -\frac{1}{2}(2t+3)^{-\frac{1}{2}-1} \frac{d}{dt}[2t+3] \\ &= -\frac{1}{2}(2t+3)^{-\frac{3}{2}} \cdot 2 \\ &= -(2t+3)^{-\frac{3}{2}}, \end{aligned}$$

and

$$
\begin{aligned}
f'''(t) &= \frac{3}{2}(2t+3)^{-\frac{3}{2}-1}\frac{d}{dt}[2t+3] \\
&= \frac{3}{2}(2t+3)^{-\frac{5}{2}} \cdot 2 \\
&= 3(2t+3)^{-\frac{5}{2}}.
\end{aligned}
$$

Therefore we have

$$
\begin{aligned}
f'''\left(\frac{1}{2}\right) &= 3(1+3)^{-\frac{5}{2}} \\
&= 3 \times 4^{-\frac{5}{2}} \\
&= 3 \times (2^2)^{-\frac{5}{2}} \\
&= 3 \times 2^{-5} \\
&= \frac{3}{2^5} \\
&= \frac{3}{32}.
\end{aligned}
$$

[30] $f'''(x) = 2\sqrt{x-1}$. Differentiating with respect to x we have

$$
\begin{aligned}
f^{(4)}(x) &= 2\frac{d}{dx}[\sqrt{x-1}] \\
&= 2\frac{d}{dx}\left[(x-1)^{\frac{1}{2}}\right] \\
&= 2 \cdot \frac{1}{2}(x-1)^{\frac{1}{2}-1}\frac{d}{dx}[x-1] \\
&= (x-1)^{-\frac{1}{2}} \\
&= \frac{1}{\sqrt{x-1}}.
\end{aligned}
$$

[36] $f(x) = (x+2)(x-2)(x+3)(x-3)$. Rewrite the function as $f(x) = (x^2-2^2)(x^2-3^2) = (x^2-4)(x^2-9)$ [Notice that I am using the formula $(a+b)(a-b) = a^2-b^2$ here]. Differentiating the function with respect to x we have

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}[x^2-4](x^2-9) + (x^2-4)\frac{d}{dx}[x^2-9] \\
&= 2x(x^2-9) + 2x(x^2-4) \\
&= 2x(x^2-9+x^2-4) \\
&= 2x(2x^2-13).
\end{aligned}
$$

Differentiating once again with respect to x we get

$$
\begin{aligned}
f''(x) &= \frac{d}{dx}[2x](2x^2-13) + 2x\frac{d}{dx}[2x^2-13] \\
&= 2(2x^2-13) + 2x \cdot 4x \\
&= 2(2x^2-13) + 8x^2 \\
&= 4x^2 - 26 + 8x^2 \\
&= 12x^2 - 26.
\end{aligned}
$$

Solving the equation $f''(x) = 0$ we have

$$
\begin{aligned}
12x^2 - 26 &= 0 \\
i.e., \quad 12x^2 = 26 \\
i.e., \quad x^2 = \frac{13}{6} \\
i.e., \quad x = \pm\sqrt{\frac{13}{6}}.
\end{aligned}
$$

Section 2.7

[3] $y^2 = 1 - x^2, \;\; 0 \le x \le 1$. Differentiating both sides with respect to x we have

$$
\begin{aligned}
\frac{d}{dx}[y^2] &= \frac{d}{dx}[1-x^2] \\
i.e., \quad 2y\frac{dy}{dx} &= -2x \\
i.e., \quad \frac{dy}{dx} &= -\frac{2x}{2y} = -\frac{x}{y}.
\end{aligned}
$$

[10] $fxy - y^2y - x = 1$. Rewrite the expression as $xy - y^2 = y - x$. Differentiating both

sides with respect to x we get

$$\begin{aligned} & \frac{d}{dx}[xy - y^2] = \frac{d}{dx}[y - x] \\ i.e., \quad & \frac{d}{dx}[xy] - 2y\frac{dy}{dx} = \frac{dy}{dx} - 1 \\ i.e., \quad & \left[x\frac{dy}{dx} + y\right] - 2y\frac{dy}{dx} = \frac{dy}{dx} - 1 \\ i.e., \quad & x\frac{dy}{dx} - 2y\frac{dy}{dx} - \frac{dy}{dx} = -1 - y \\ i.e., \quad & (x - 2y - 1)\frac{dy}{dx} = -(1 + y) \\ i.e., \quad & \frac{dy}{dx} = -\frac{(1 + y)}{x - 2y - 1}. \end{aligned}$$

[21] $x^{1/2} + y^{1/2} = 9$. Differentiating both sides with respect to x we get

$$\begin{aligned} & \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0 \\ i.e., \quad & x^{-\frac{1}{2}} + y^{-\frac{1}{2}}\frac{dy}{dx} = 0 \\ i.e., \quad & y^{-\frac{1}{2}}\frac{dy}{dx} = -x^{-\frac{1}{2}} \\ i.e., \quad & \frac{dy}{dx} = -\frac{x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}} \\ i.e., \quad & \frac{dy}{dx} = -\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}. \end{aligned}$$

Therefore

$$\begin{aligned} \left.\frac{dy}{dx}\right|_{(16,25)} &= \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}. \\ &= \frac{5}{4}. \end{aligned}$$

[24] $(x+y)^3 = x^3 + y^3$. Differentiating both side with respect to x we have

$$
\begin{aligned}
& 3(x+y)^2\frac{d}{dx}[x+y] = 3x^2 + 3y^2\frac{d}{dx}[y] \\
i.e., \quad & 3(x+y)^2\left[1+\frac{dy}{dx}\right] = 3x^2 + 3y^2\frac{dy}{dx} \\
i.e., \quad & (x+y)^2\left[1+\frac{dy}{dx}\right] = x^2 + y^2\frac{dy}{dx} \\
i.e., \quad & (x+y)^2 + (x+y)^2\frac{dy}{dx} = x^2 + y^2\frac{dy}{dx} \\
i.e., \quad & (x+y)^2\frac{dy}{dx} - y^2\frac{dy}{dx} = x^2 - (x+y)^2 \\
i.e., \quad & [(x+y)^2 - y^2]\frac{dy}{dx} = x^2 - (x^2 + 2xy + y^2) \\
i.e., \quad & [(x^2 + 2xy + y^2) - y^2]\frac{dy}{dx} = x^2 - x^2 - 2xy - y^2 \\
i.e., \quad & [x^2 + 2xy]\frac{dy}{dx} = -2xy - y^2 \\
i.e., \quad & \frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy} \\
i.e., \quad & \frac{dy}{dx} = -\frac{y(2x+y)}{x(x+2y)}.
\end{aligned}
$$

[29] $4x^2 + 9y^2 = 36$. Differentiating both sides with respect to x we have

$$
\begin{aligned}
& 8x + 18y\frac{dy}{dx} = 0 \\
i.e., \quad & 18y\frac{dy}{dx} = -8x \\
i.e., \quad & \frac{dy}{dx} = -\frac{8x}{18y} = -\frac{4x}{9y}.
\end{aligned}
$$

Therefore slope of the tangent line at $\left(\sqrt{5}, \frac{4}{3}\right)$ is

$$
\begin{aligned}
\left.\frac{dy}{dx}\right|_{\left(\sqrt{5},\frac{4}{3}\right)} &= -\frac{4\sqrt{5}}{9\cdot\frac{4}{3}} \\
&= -\frac{4\sqrt{5}}{12} \\
&= -\frac{\sqrt{5}}{3}.
\end{aligned}
$$

[37] Given equation $y^2 = 5x^3$. Differentiating both sides with respect to x we have

$$2y\frac{dy}{dx} = 15x^2$$
$$i.e., \qquad \frac{dy}{dx} = \frac{15x^2}{2y}.$$

Therefore slope of the tangent line at $(1, \sqrt{5})$ is

$$\left.\frac{dy}{dx}\right|_{(1,\sqrt{5})} = \frac{15}{2\sqrt{5}}.$$

Equation of the tangent line at $(1, \sqrt{5})$ is

$$y - \sqrt{5} = \frac{15}{2\sqrt{5}}(x-1)$$
$$i.e., \qquad 2\sqrt{5}(y - \sqrt{5}) = 15(x-1)$$
$$i.e., \qquad 2\sqrt{5}y - 10 = 15x - 15$$
$$i.e., \qquad 2\sqrt{5}y - 15x + 5 = 0.$$

Similarly, slope of the tangent line at $(1, -\sqrt{5})$ is

$$\left.\frac{dy}{dx}\right|_{(1,-\sqrt{5})} = -\frac{15}{2\sqrt{5}}.$$

Equation of the tangent line at $(1, -\sqrt{5})$ is

$$y - \sqrt{5} = -\frac{15}{2\sqrt{5}}(x-1)$$
$$i.e., \qquad 2\sqrt{5}(y - \sqrt{5}) = -15(x-1)$$
$$i.e., \qquad 2\sqrt{5}y - 10 = -15x + 15$$
$$i.e., \qquad 2\sqrt{5}y + 15x - 25 = 0.$$

[43] We have to find the rate of change of x with respect to p i.e., $\frac{dx}{dp}$. Let us rewrite the given equation in the following form

$$p^2 = \frac{200 - x}{2x}$$
$$i.e., \qquad 2xp^2 = 200 - x.$$

Differentiating both sides with respect to p we have

$$\frac{d}{dp}[2xp^2] = \frac{d}{dp}[200 - x]$$

$$i.e., \qquad \frac{d}{dp}[2x]p^2 + 2x\frac{d}{dp}[p^2] = -\frac{dx}{dp}$$

$$i.e., \qquad 2p^2\frac{dx}{dp} + 4xp = -\frac{dx}{dp}$$

$$i.e., \qquad 2p^2\frac{dx}{dp} + \frac{dx}{dp} = -4xp$$

$$i.e., \qquad (2p^2 + 1)\frac{dx}{dp} = -4xp$$

$$i.e., \qquad \frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}.$$