Section 2.5

[4] Given function $y = (x^2+1)^{4/3}$. Inside function is $u = g(x) = x^2+1$, and outside function is $y = f(u) = u^{4/3}$.

[6] Given function $y = \sqrt{9-x^2}$. Inside function is $u = g(x) = 9-x^2$, and outside function is $y = f(u) = \sqrt{u}$.

[14] $f(x) = \frac{x^4-2x+1}{\sqrt{x}}$. At first look, we are tempted to use quotient rule. But it can be solved efficiently using simple power rule only. Rewrite $f(x) = (x^4-2x+1)x^{-1/2} = x^{7/2} - 2x^{1/2} + x^{-1/2}$. Now using power rule we get

$$\begin{aligned} f'(x) &= \frac{7}{2}x^{5/2} - 2\frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} \\ &= \frac{7}{2}x^{5/2} - \frac{1}{x^{1/2}} - \frac{1}{2x^{3/2}}. \end{aligned}$$

[Note: we can also use quotient rule. But that would take longer time].

[27] $s(t) = \sqrt{2t^2+5t+2}$. Take $f(t) = 2t^2+5t+2$. Then we have $s(t) = [f(t)]^{1/2}$. Using general power rule we have

$$\begin{aligned} s'(t) &= \frac{1}{2}[f(t)]^{-1/2}f'(t) \\ &= \frac{1}{2}[f(t)]^{-1/2}(4t+5+0) \\ &= \frac{1}{2}(2t^2+5t+2)^{-1/2}(4t+5) \\ &= \frac{4t+5}{2\sqrt{2t^2+5t+2}}. \end{aligned}$$

[28] $y = \sqrt[3]{3x^3+4x}$. Take $f(x) = 3x^3+4x$. Then we have $y = [f(x)]^{1/3}$. Using general

power rule we have

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{1}{3}[f(x)]^{-2/3} f'(x) \\
&= \frac{1}{3}[f(x)]^{-2/3}(9x^2+4) \\
&= \frac{1}{3}(x^3+4x)^{-2/3}(9x^2+4) \\
&= \frac{9x^2+4}{3(x^3+4x)^{2/3}}.
\end{aligned}
$$

[38] $f(x) = x\sqrt{x^2+5}$. Using product rule we have

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}[x]\sqrt{x^2+5} + x\frac{d}{dx}[\sqrt{x^2+5}] \\
&= \sqrt{x^2+5} + x\left[\frac{1}{2}(x^2+5)^{-1/2}\cdot(2x+0)\right] \qquad \text{(chain rule/general power rule)} \\
&= \sqrt{x^2+5} + \frac{x^2}{\sqrt{x^2+5}}.
\end{aligned}
$$

Therefore slope of the tangent line is $f'(2) = \sqrt{4+5} + \frac{4}{\sqrt{4+5}} = 3 + \frac{4}{3} = \frac{13}{3}$. Notice that $f(2) = 2\sqrt{4+5} = 6$. Hence equation of the tangent line is

$$
\begin{aligned}
& y - f(2) = f'(2)(x-2) \\
i.e, \quad & y - 6 = \frac{13}{4}(x-2) \\
i.e, \quad & 4y - 24 = 13x - 26 \\
i.e, \quad & 4y - 13x + 2 = 0.
\end{aligned}
$$

[61] $f(x) = \sqrt{x^2+1} - \sqrt{x^2-1}$. Using chain rule (or general power rule) we have

$$
\begin{aligned}
f'(x) &= \frac{1}{2}(x^2+1)^{-1/2}\frac{d}{dx}[x^2+1] - \frac{1}{2}(x^2-1)^{-1/2}\frac{d}{dx}[x^2-1] \\
&= \frac{1}{2}(x^2+1)^{-1/2}(2x+0) - \frac{1}{2}(x^2-1)^{-1/2}(2x+0) \\
&= \frac{x}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2-1}}.
\end{aligned}
$$

[64] $y = \left(\frac{4x^2}{3-x}\right)^3$. Using chain rule (or general power rule) we have

$$\begin{aligned}
\frac{dy}{dx} &= 3\left(\frac{4x^2}{3-x}\right)^2 \frac{d}{x}\left[\frac{4x^2}{3-x}\right] \\
&= 3\left(\frac{4x^2}{3-x}\right)^2 \left[\frac{(3-x)\frac{d}{dx}[4x^2] - 4x^2\frac{d}{dx}[3-x]}{(3-x)^2}\right] \qquad \text{(quotient rule)} \\
&= 3\left(\frac{4x^2}{3-x}\right)^2 \left[\frac{8x(3-x) - 4x^2(0-1)}{(3-x)^2}\right] \\
&= 3\left(\frac{4x^2}{3-x}\right)^2 \left[\frac{24x - 8x^2 + 4x^2}{(3-x)^2}\right] \\
&= 3\left(\frac{4x^2}{3-x}\right)^2 \left[\frac{24x - 4x^2}{(3-x)^2}\right] \\
&= 3\left(\frac{4x^2}{3-x}\right)^2 \left[\frac{4x(6-x)}{(3-x)^2}\right] \\
&= 12x\left(\frac{4x^2}{3-x}\right)^2 \frac{6-x}{(3-x)^2}.
\end{aligned}$$

[66] $s(x) = \frac{1}{\sqrt{x^2-3x+4}}$. We can rewrite $s(x) = (x^2-3x+4)^{-1/2}$. Using chain rule (or general power rule) we have

$$\begin{aligned}
s'(x) &= -\frac{1}{2}(x^2-3x+4)^{-3/2}\frac{d}{dx}[x^2-3x+4] \\
&= -\frac{1}{2}(x^2-3x+4)^{-3/2}[2x-3] \\
&= -\frac{2x-3}{2(x^2-3x+4)^{3/2}}.
\end{aligned}$$

Therefore slope of the tangent line is $s'(3) = -\frac{6-3}{2(9-9+4)^{3/2}} = -\frac{3}{2\times\sqrt[2]{4^3}} = -\frac{3}{16}$. Hence equation of the tangent line at $\left(3, \frac{1}{2}\right)$ is

$$\begin{aligned}
&y - \frac{1}{2} = -\frac{3}{16}(x-3) \\
&16y - 8 = -3x + 9 \\
&16y + 3x - 17 = 0.
\end{aligned}$$

Section 8.4

[22] $y = \tan e^x$. Using chain rule we have

$$\begin{aligned}\frac{dy}{dx} &= \sec^2 e^x \frac{d}{dx}[e^x] \\ &= e^x \sec^2 e^x.\end{aligned}$$

[24] $y = -\sin^4 2x$. Using chain rule we have

$$\begin{aligned}\frac{dy}{dx} &= -4\sin^3 2x \frac{d}{dx}[\sin 2x] \\ &= -4\sin^3 2x \cos 2x \frac{d}{dx}[2x] \\ &= -8\sin^3 2x \cos 2x.\end{aligned}$$

[25] $y = e^{2x}\sin 2x$. Using product rule and chain rule we have

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[e^{2x}]\sin 2x + e^{2x}\frac{d}{dx}[\sin 2x] \\ &= e^{2x}\frac{d}{dx}[2x]\sin 2x + e^{2x}\cos 2x \frac{d}{dx}[2x] \\ &= 2e^{2x}\sin 2x + 2e^{2x}\cos 2x \\ &= 2e^{2x}(\sin 2x + \cos 2x).\end{aligned}$$

[36] $y = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5}$. Using chain rule (or general power rule) we have

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{7}7\sec^6 x \frac{d}{dx}[\sec x] - \frac{1}{5}5\sec^4 x \frac{d}{dx}[\sec x] \\ &= \sec^6 x[\sec x \tan x] - \sec^4 x[\sec x \tan x] \\ &= \sec^7 x \tan x - \sec^5 x \tan x \\ &= (\sec^2 x - 1)\sec^5 x \tan x \\ &= \tan^2 x \sec^5 x \tan x \quad (\text{since } \sec^2 x - \tan^2 x = 1). \\ &= \sec^5 x \tan^3 x.\end{aligned}$$