
Finding derivatives:

1. $f(x) = \frac{\sin x}{x}$. Using quotient rule we obtain

$$\begin{aligned} f'(x) &= \frac{x\frac{d}{dx}[\sin x] - \sin x\frac{d}{dx}[x]}{\sin^2 x} \\ &= \frac{x\cos x - \sin x}{\sin^2 x}. \end{aligned}$$

2. $f(x) = x^2 \sec x$. Using product rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[x^2]\sec x + x^2\frac{d}{dx}[\sec x] \\ &= 2x\sec x + x^2\sec x\tan x \\ &= x\sec x(2 + x\tan x). \end{aligned}$$

3. $f(x) = \sin x \tan x$. Using product rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[\sin x]\tan x + \sin x\frac{d}{dx}[\tan x] \\ &= \cos x\tan x + \sin x\sec^2 x \\ &= \cos x\frac{\sin x}{\cos x} + \sin x\sec^2 x \\ &= \sin x + \sin x\sec^2 x \\ &= (1 + \sec^2 x)\sin x. \end{aligned}$$

4. $f(x) = e^x \sec x$. Using product rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[e^x]\sec x + e^x\frac{d}{dx}[\sec x] \\ &= e^x\sec x + e^x\sec x\tan x \\ &= e^x(1 + \tan x)\sec x. \end{aligned}$$

5. $f(x) = \frac{\cot x}{1+x^2}$. Using quotient rule we have

$$\begin{aligned} f'(x) &= \frac{(1+x^2)\frac{d}{dx}[\cot x] - \cot x\frac{d}{dx}[1+x^2]}{(1+x^2)^2} \\ &= \frac{-(1+x^2)\csc^2 x - (\cot x)(0+2x)}{(1+x^2)^2} \\ &= -\frac{(1+x^2)\csc^2 x + 2x\cot x}{(1+x^2)^2}. \end{aligned}$$

6. $f(x) = xe^x \csc x$. Using general product rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[x]e^x \csc x + x\frac{d}{dx}[e^x] \csc x + xe^x \frac{d}{dx}[\csc x] \\ &= e^x \csc x + xe^x \csc x - xe^x \csc x \cot x \\ &= e^x(1 + x - x \cot x) \csc x. \end{aligned}$$

Equation of tangent line:

1. $f(x) = e^x \sin x$. Using the product rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[e^x] \sin x + e^x \frac{d}{dx}[\sin x] \\ &= e^x \sin x + e^x \cos x \\ &= e^x(\sin x + \cos x). \end{aligned}$$

Slope of the tangent line at $x = 0$ is $f'(0) = e^0(\sin 0 + \cos 0) = 1$. Also notice that $f(0) = e^0 \sin 0 = 0$. Therefore equation of the tangent line at $x = 0$ is

$$\begin{aligned} & y - f(0) = f'(0)(x - 0) \\ i.e., \quad & y = x. \end{aligned}$$

2. $f(t) = (1 + t^2) \cos t$. Using product rule we have

$$\begin{aligned} f'(t) &= \frac{d}{dt}[1 + t^2] \cos t + (1 + t^2)\frac{d}{dt}[\cos t] \\ &= 2t \cos t - (1 + t^2) \sin t. \end{aligned}$$

Therefore slope of the tangent line at $x = \frac{\pi}{2}$ is

$$\begin{aligned} f'\left(\frac{\pi}{2}\right) &= 2\frac{\pi}{2} \cos \frac{\pi}{2} - \left(1 + \frac{\pi^2}{4}\right) \sin \frac{\pi}{2} \\ &= -\left(1 + \frac{\pi^2}{4}\right). \end{aligned}$$

Notice that $f\left(\frac{\pi}{2}\right) = \left(1 + \frac{\pi^2}{4}\right) \cos \frac{\pi}{2} = 0$. Therefore equation of the tangent line at $x = \frac{\pi}{2}$ is

$$\begin{aligned} & y - f\left(\frac{\pi}{2}\right) = f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) \\ i.e., \quad & y = -\left(1 + \frac{\pi^2}{4}\right)\left(x - \frac{\pi}{2}\right). \end{aligned}$$

3. $g(x) = e^2 \tan x$. Since e^2 is just a constant, it is easy to see that $g'(x) = e^2 \sec^2 x$. Therefore slope of the tangent line at $x = \frac{\pi}{4}$ is $f'\left(\frac{\pi}{4}\right) = e^2 \sec^2 \frac{\pi}{4} = 2e^2$. Therefore equation of the tangent line at $\left(\frac{\pi}{4}, e^2\right)$ is

$$\begin{aligned} & y - e^2 = 2e^2\left(x - \frac{\pi}{4}\right) \\ i.e., \qquad & y - 2e^2 x - e^2\left(1 - \frac{\pi}{2}\right) = 0. \end{aligned}$$

4. $f(x) = xe^x \sec x$. Using the general product rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[x]e^x \sec x + x\frac{d}{dx}[e^x]\sec x + xe^x\frac{d}{dx}[\sec x] \\ &= e^x \sec x + xe^x \sec x + xe^x \sec x \tan x \\ &= e^x(1 + x + x\tan x)\sec x. \end{aligned}$$

Therefore slope of the tangent line at $(0, 0)$ is

$$\begin{aligned} f'(0) &= e^0(1 + 0 + 0\tan 0)\sec 0 \\ &= 1. \end{aligned}$$

Hence equation of the tangent line at $(0, 0)$ is

$$\begin{aligned} & y - 0 = (x - 0) \\ i.e, \qquad & y - x = 0. \end{aligned}$$