
Section 2.2

[1]

(a) $f(x) = x^2$. Using power rule we have $f'(x) = 2x$. Therefore slope of the tangent at $x = 1$ is $f'(1) = 2$.

(b) $f(x) = x^{1/2}$. Using power rule we have $f'(x) = \frac{1}{2}x^{-1/2}$. Therefore slope of the tangent at $x = 1$ is $f'(1) = \frac{1}{2}$.

[4]

(a) $f(x) = x^{-1/2}$. Using power rule we have $f'(x) = -\frac{1}{2}x^{-3/2}$. Therefore slope of the tangent at $x = 1$ is $f'(1) = -\frac{1}{2}$.

(b) $f(x) = x^{-2}$. Using power rule we have $f'(x) = -2x^{-3}$. Therefore slope of the tangent at $x = 1$ is $f'(1) = -2$.

[6] $f(x) = -2$. Using constant rule we have $f'(x) = 0$.

[12] $f(x) = x^3 - 9x^2 + 2$. Using sum-difference rule, constant multiple rule, power rule, and constant rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[x^3] - \frac{d}{dx}[9x^2] + \frac{d}{dx}[2] \\ &= 3x^2 - 9\frac{d}{dx}[x^2] + 0 \\ &= 3x^2 - 18x \end{aligned}$$

[15] $f(t) = 4t^{4/3}$. Using power rule, and constant multiple rule we have $f'(t) = 4 \times \frac{4}{3} t^{\frac{4}{3}-1} = \frac{16}{3}t^{1/3}$.

[18] $g(x) = 4\sqrt[3]{x} + 2$. Using sum-difference rule, constant multiple rule, constant rule, and

power rule we have

$$\begin{aligned} g'(x) &= 4\frac{d}{dx}[\sqrt[3]{x}] + 0 \\ &= 4\frac{d}{dx}[x^{1/3}] \\ &= 4\frac{1}{3}x^{1/3-1} \\ &= \frac{4}{3}x^{-2/3} \end{aligned}$$

[23] $f(x) = \frac{1}{(4x)^3}$. we rewrite it as $f(x) = \frac{1}{4^3x^3} = \frac{1}{4^3}x^{-3}$. Using power rule we have $f'(x) = \frac{1}{4^3} \times (-3)x^{-4}$. After simplification we have $f'(x) = -\frac{3}{64x^4}$.

[25] $f(x) = \frac{\sqrt{x}}{x}$. Rewrite it as $f(x) = \frac{x^{1/2}}{x} = x^{-1/2}$. Using power rule we have $f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$. After simplification we have $f'(x) = -\frac{1}{2x^{3/2}}$.

[26] $f(x) = \frac{4x}{x^{-3}}$. Rewrite it as $f(x) = 4x \times x^3 = 4x^4$. Using power rule we have $f'(x) = 4 \times 4x^3 = 16x^3$. No further simplification is needed.

[29] $f(x) = -\frac{1}{2}x(1+x^2)$. Using product rule and power rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[-\frac{x}{2}\right](1+x^2) + \left[-\frac{x}{2}\right]\frac{d}{dx}[(1+x^2)] \\ &= -\frac{1}{2}(1+x^2) - \frac{x}{2}(0+2x) \\ &= -\frac{1}{2}(1+x^2) - x^2 \\ &= -\frac{1}{2} - \frac{3}{2}x^2. \end{aligned}$$

Therefore value of the derivative at $(1, -1)$ is $f'(1) = -\frac{1}{2} - \frac{3}{2} = -2$.

[31] $f(x) = (2x+1)^2$. Using power rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[(2x+1)^2] \\ &= \frac{d}{dx}[4x^2 + 4x + 1] \\ &= (4 \times 2x) + 4 \\ &= 8x + 4. \end{aligned}$$

Therefore the value of the derivative at $(0, 1)$ is $f'(0) = 4$.

This problem can also be solved using the chain rule. We will discuss about that in next lecture.

[41] $f(x) = \frac{2x^3-4x^2+3}{x^2}$. Using quotient rule we obtain

$$\begin{aligned} f'(x) &= \frac{x^2\frac{d}{dx}[2x^3-4x^2+3]-(2x^3-4x^2+3)\frac{d}{dx}[x^2]}{x^4} \\ &= \frac{x^2[6x^2-8x]-(2x^3-4x^2+3)\times 2x}{x^4} \\ &= \frac{[6x^4-8x^3]-[4x^4-8x^3+6x]}{x^4} \\ &= \frac{2x^4-6x}{x^4} \\ &= 2-\frac{6}{x^3}. \end{aligned}$$

Alternative method: We can rewrite the function as $f(x) = 2x-4+3x^{-2}$. Now using power rule we have

$$f'(x) = 2+3\times(-2)x^{-3} = 2-\frac{6}{x^3}.$$

[44] $f(x) = \frac{-6x^3+3x^2-2x+1}{x}$. We can rewrite the function as $f(x) = -6x^2+3x-2+x^{-1}$. Using the power rule we have

$$f'(x) = -12x+3-x^{-2}.$$

[49] $f(x) = \sqrt[3]{x}+\sqrt[5]{x}$. Using power rule we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}[\sqrt[3]{x}+\sqrt[5]{x}] \\ &= \frac{d}{dx}[x^{1/3}+x^{1/5}] \\ &= \frac{1}{3}x^{\frac{1}{3}-1}+\frac{1}{5}x^{\frac{1}{5}-1} \\ &= \frac{1}{3}x^{-2/3}+\frac{1}{5}x^{-4/5} \\ &= \frac{1}{3x^{2/3}}+\frac{1}{5x^{4/5}}. \end{aligned}$$

Therefore slope of the tangent line is $f'(1) = \frac{1}{3}+\frac{1}{5} = \frac{8}{15}$. Hence equation of the tangent line at $(1,2)$ is

$$\begin{aligned} y-2 &= \frac{8}{15}(x-1) \\ \textit{i.e.,} \quad 15y-30 &= 8x-8 \\ \textit{i.e.,} \quad 15y-8x-22 &= 0. \end{aligned}$$

[53] $f(x) = \frac{1}{2}x^2 + 5x$. Using power rule we have $f'(x) = \frac{1}{2}2x + 5 = x + 5$. We know that slope of the tangent line at x is $f'(x)$. If the tangent line is horizontal, then slope of the tangent line must be zero. Solving the equation $f'(x) = 0$ we obtain $x = -5$. Therefore at $x = -5$ we have $f'(x) = 0$. Hence at $x = -5$ the graph of $f(x)$ has a horizontal tangent line.

[57(a)] Given $f(x) = h(x) - 2$. Differentiating both sides with respect to x we get $f'(x) = h'(x) - 0 = h'(x)$. Since we know that $f'(1) = 3$, therefore we have $h'(1) = f'(1) = 3$

[57(d)] Given $h(x) = -1 + 2f(x)$. Differentiating both sides with respect to x we get $h'(x) = 0 + 2f'(x) = 2f'(x)$. Since $f'(1) = 3$, we get $h'(1) = 2f'(1) = 6$.

Section 2.4

[5] $g(x) = (x^2 - 4x + 3)(x - 2)$. Using product rule

$$\begin{aligned} g'(x) &= \frac{d}{dx}[x^2 - 4x + 3](x - 2) + (x^2 - 4x + 3)\frac{d}{dx}[x - 2] \\ &= (2x - 4)(x - 2) + (x^2 - 4x + 3)(1 + 0) \\ &= (2x - 4)(x - 2) + (x^2 - 4x + 3). \end{aligned}$$

Therefore $g'(4) = (8 - 4)(4 - 2) + (16 - 16 + 3) = 8 + 3 = 11$.

Alternative method: Rewrite the function as $g(x) = (x - 1)(x - 3)(x - 2)$. Using general product rule we get

$$\begin{aligned} g'(x) &= \left[\frac{d}{dx}(x - 1)\right](x - 3)(x - 2) + (x - 1)\left[\frac{d}{dx}(x - 3)\right](x - 2) + (x - 1)(x - 3)\left[\frac{d}{dx}(x - 2)\right] \\ &= (1 + 0)(x - 3)(x - 2) + (x - 1)(1 + 0)(x - 2) + (x - 1)(x - 3)(1 + 0) \\ &= (x - 3)(x - 2) + (x - 1)(x - 2) + (x - 1)(x - 3). \end{aligned}$$

Therefore $g'(4) = (4 - 3)(4 - 2) + (4 - 1)(4 - 2) + (4 - 1)(4 - 3) = 2 + 6 + 3 = 11$.

[19] $f(x) = \frac{4x^2 - 3x}{8\sqrt{x}}$. Rewrite the function as

$$\begin{aligned} f(x) &= \frac{4x^2 - 3x}{8x^{1/2}} \\ &= \frac{1}{8}(4x^2 - 3x)x^{-1/2} \\ &= \frac{1}{8}(4x^{3/2} - 3x^{1/2}) \\ &= \frac{1}{2}x^{3/2} - \frac{3}{8}x^{1/2}. \end{aligned}$$

Using power rule we obtain

$$
\begin{aligned}
f'(x) &= \left[\frac{1}{2}\times\frac{3}{2}x^{1/2}\right] - \left[\frac{3}{8}\times\frac{1}{2}x^{-1/2}\right] \\
&= \frac{3}{4}x^{1/2} - \frac{3}{16}x^{-1/2} \\
&= \frac{3}{4}\sqrt{x} - \frac{3}{16\sqrt{x}}
\end{aligned}
$$

[21] $f(x) = \frac{x^2-4x+3}{x-1}$. Note that the function is not defined for $x = 1$ (we have $\frac{0}{0}$ form for $x = 1$). We can rewrite the function as $f(x) = \frac{(x-1)(x-3)}{x-1} = x - 3$ for $x \neq 1$. Differentiating, we get $f'(x) = 1 + 0 = 1$ for $x \neq 1$.

[31] $f(x) = \frac{3-2x-x^2}{x^2-1}$. This function is not defined for $x = \pm 1$. We can rewrite the function as $f(x) = \frac{(1-x)(3+x)}{(x+1)(x-1)} = -\frac{3+x}{x+1}$ if $x \neq \pm 1$. Using quotient rule we have

$$
\begin{aligned}
f'(x) &= -\frac{(x+1)\left[\frac{d}{dx}(3+x)\right] - (3+x)\left[\frac{d}{dx}(x+1)\right]}{(}x+1)^2 \\
&= -\frac{(x+1)-(3+x)}{(x+1)^2} \\
&= -\frac{-2}{(x+1)^2} \\
&= \frac{2}{(x+1)^2},
\end{aligned}
$$

for $x \neq \pm 1$.

[37] $g(x) = \left(\frac{x-3}{x+4}\right)(x^2+2x+1)$. Using product rule and quotient rule we have

$$\begin{aligned}
g'(x) &= \left[\frac{d}{dx}\left(\frac{x-3}{x+4}\right)\right](x^2+2x+1) + \left(\frac{x-3}{x+4}\right)\left[\frac{d}{dx}(x^2+2x+1)\right] \\
&= \left[\frac{(x+4)\frac{d}{dx}(x-3)-(x-3)\frac{d}{dx}(x+4)}{(x+4)^2}\right](x^2+2x+1) + \left(\frac{x-3}{x+4}\right)[2x+2] \\
&= \left[\frac{(x+4)-(x-3)}{(x+4)^2}\right](x^2+2x+1) + 2\left(\frac{x-3}{x+4}\right)(x+1) \\
&= \frac{7}{(x+4)^2}(x^2+2x+1) + 2\left(\frac{x-3}{x+4}\right)(x+1) \\
&= \frac{7(x+1)^2}{(x+4)^2} + 2\left(\frac{x-3}{x+4}\right)(x+1) \\
&= \frac{(x+1)}{(x+4)}\left[\frac{7(x+1)}{(x+4)} + 2(x-3)\right] \\
&= \frac{(x+1)}{(x+4)}\left[\frac{7(x+1)+2(x-3)(x+4)}{(x+4)}\right] \\
&= \frac{(x+1)}{(x+4)}\left[\frac{7x+7+2(x^2+x-12)}{(x+4)}\right] \\
&= \frac{(x+1)}{(x+4)}\frac{(2x^2+9x-17)}{(x+4)} \\
&= \frac{(2x^3+9x^2-17x)+(2x^2+9x-17)}{(x+4)^2} \\
&= \frac{2x^3+11x^2-8x-17}{(x+4)^2}.
\end{aligned}$$

[43] $f(x) = \left(\frac{x+5}{x-1}\right)(2x+1)$. Using product rule and quotient rule we have

$$\begin{aligned}
f'(x) &= \left[\frac{d}{dx}\left(\frac{x+5}{x-1}\right)\right](2x+1) + \left(\frac{x+5}{x-1}\right)\left[\frac{d}{dx}(2x+1)\right] \\
&= \left[\frac{(x-1)\frac{d}{dx}(x+5)-(x+5)\frac{d}{dx}(x-1)}{(x-1)^2}\right](2x+1) + \left(\frac{x+5}{x-1}\right)[2+0] \\
&= \left[\frac{(x-1)-(x+5)}{(x-1)^2}\right](2x+1) + 2\left(\frac{x+5}{x-1}\right) \\
&= \frac{-6(2x+1)}{(x-1)^2} + 2\left(\frac{x+5}{x-1}\right).
\end{aligned}$$

Slope of the tangent line at $x = 0$ is

$$
\begin{aligned}
f'(0) &= \frac{-6}{(-1)^2} + 2\frac{5}{-1} \\
&= -6 - 10 \\
&= -16.
\end{aligned}
$$

Therefore equation of the tangent line at $(0, -5)$ is

$$
\begin{aligned}
& y - (-5) = -16(x - 0) \\
i.e., \quad & y + 16x + 5 = 0.
\end{aligned}
$$