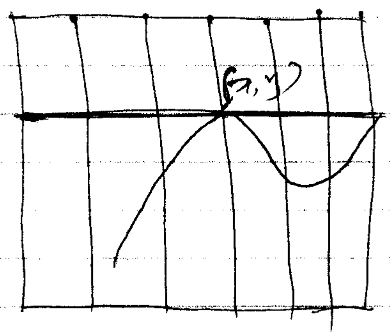
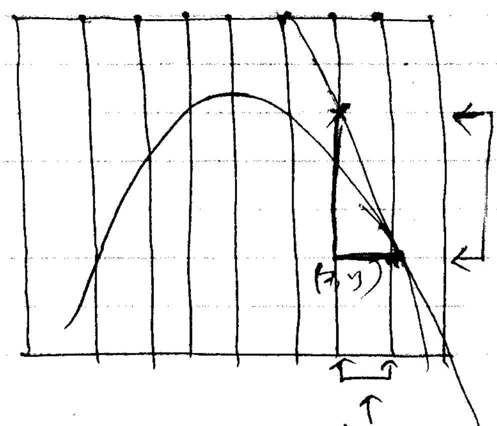
Exercises 2.1 (page 90)

7)



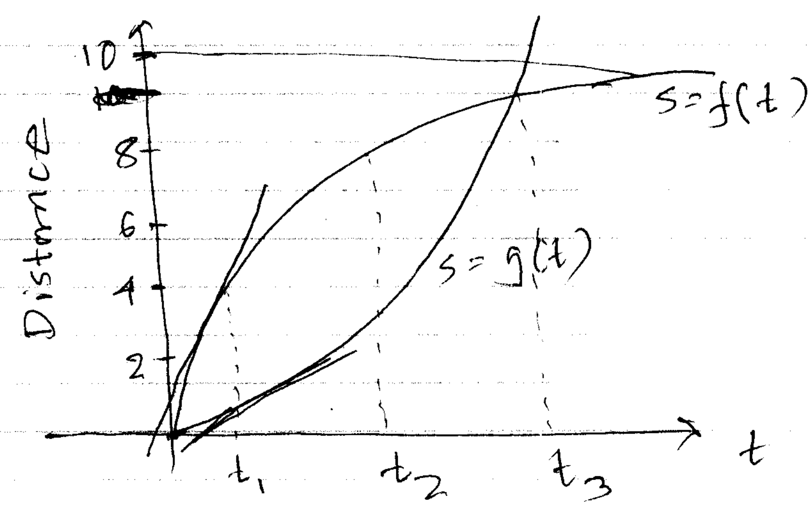
Since the tangent line is a horizontal line, the slope of the graph at (7, y) is 0 (zero).

10)



Therefore slope $= \frac{3}{-1} = -3$.

14)

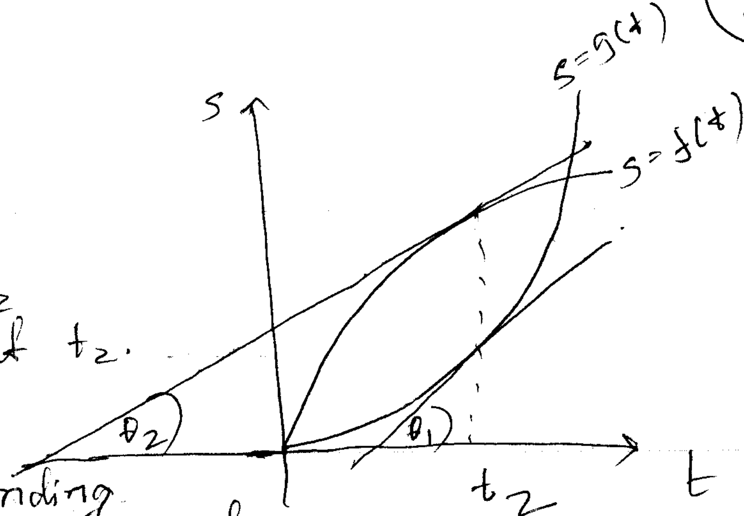


ⓐ Note that $s = f(t)$ has higher slope than $s = g(t)$ at t_1. Therefore the runner corresponding to $s = f(t)$ is going faster than the runner corresponding to $s = g(t)$ at t_1.

(b)

since $\theta_1 > \theta_2$,

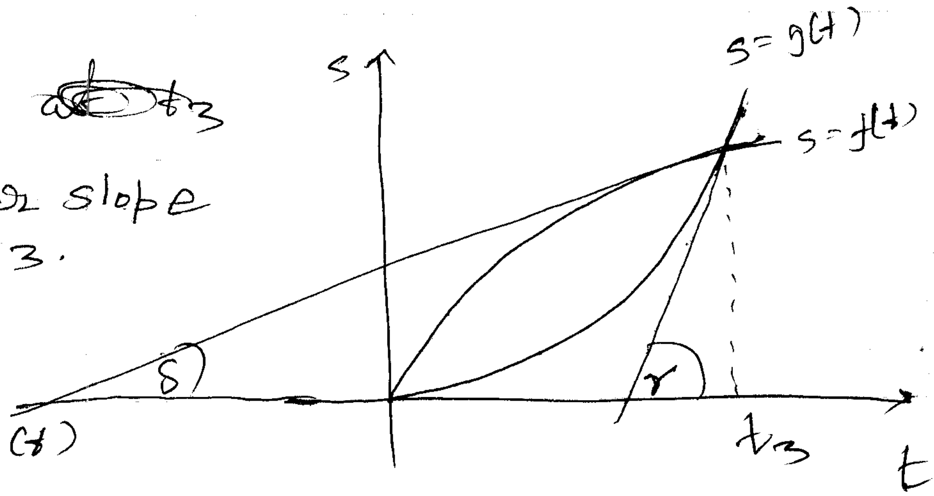
$\therefore$ $s = g(t)$ has greater slope than $s = f(t)$ at t_2.



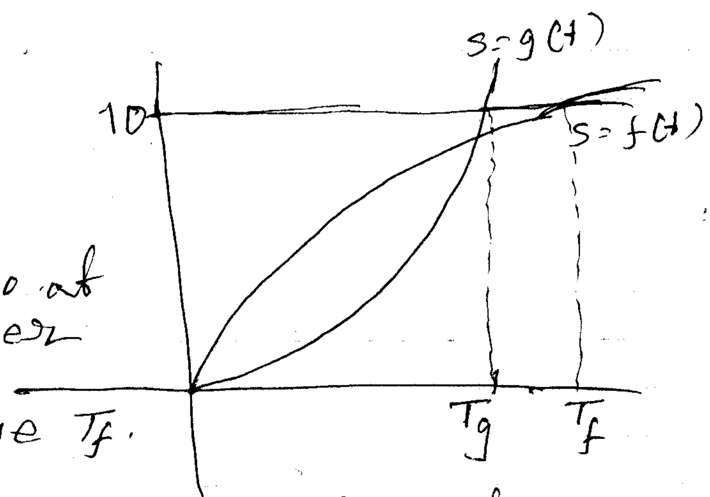
So the runner corresponding to $s = g(t)$ is going faster at t_2

(c) since $\gamma > \delta$, ~~at t_3~~

$s = g(t)$ has greater slope than $s = f(t)$ at t_3.



$\therefore$ The runner corresponding to $s = g(t)$ is going faster at t_3.



(d) ~~Rotor~~ ~~dote~~

Runner g reaches 10,000 at time T_g, whereas runner f reaches 10,000 at the time T_f.

Since $T_f > T_g$, the runner g finishes the race first.

22)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{(x+h)+2} - \sqrt{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{(x+h)+2} - \sqrt{x+2}\right)\left(\sqrt{(x+h)+2} + \sqrt{x+2}\right)}{h\left(\sqrt{(x+h)+2} + \sqrt{x+2}\right)}$$

$$= \lim_{h\to 0} \frac{[(x+h)+2] - [x+2]}{h\left(\sqrt{(x+h)+2} + \sqrt{x+2}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+2} + \sqrt{x+2}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}}$$

$$= \frac{1}{2\sqrt{x+2}}.$$

25) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{(x+2) - (x+h+2)}{(x+h+2)(x+2)h}$$

$$= \lim_{h\to 0} \frac{-h}{(x+h+2)(x+2)h} = -\frac{1}{(x+2)^2}.$$

37)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2}{2} - \frac{x^2}{2}}{h}$$

$$= \lim_{h\to 0} \frac{(x^2+2xh+h^2) - x^2}{2h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{2h}$$

$$= \lim_{h\to 0} \frac{1}{2}(2x+h) = x.$$

$\therefore$ Slope of the tangent line is $f'(2) = 2$.

Equation of the tangent line is,

$$y - 2 = f'(2)(x-2)$$

i.e $y - 2 = 2(x-2)$

i.e $y - 2x + 2 = 0$.

44) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{(x-1) - (x+h-1)}{(x+h-1)(x-1)h}$$

$$= \lim_{h\to 0} \frac{-h}{(x+h-1)(x-1)h} = -\frac{1}{(x-1)^2}.$$

$\therefore$ Slope of the tangent line at $x = 2$ is

$$f'(2) = -\frac{1}{(2-1)^2} = -1.$$

$\therefore$ Equation of the tangent line at $(2,1)$ is

$$y - 1 = f'(2)(x-2)$$

i.e. $y - 1 = -(x-2)$

i.e. $y + x - 3 = 0$.

45) Done in class

48) Slope of the line $x+2y-6=0$ is $-\frac{1}{2}$.

Because we can write it as

$$y = -\frac{1}{2}x + 3.$$

Since the tangent line is parallel to the line $x+2y-6=0$, slope of the tangent line is also $-\frac{1}{2}$.

We notice that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^2 - (x+h)] - [x^2 - x]}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{x^2} + 2hx + h^2 - \cancel{x} - h) - (\cancel{x^2} - \cancel{x})}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2 - h}{h}$$

$$= \lim_{h \to 0} (2x + h - 1) = 2x - 1$$

We know that slope of the tangent line at (x, y) is $f'(x)$.

$$\therefore \quad f'(x) = -\frac{1}{2}$$

$$\Rightarrow 2x - 1 = -\frac{1}{2}$$

$$\Rightarrow 2x = 1 - \frac{1}{2}$$

$$\Rightarrow 2x = \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{4}.$$

$$\therefore \quad y = f(x) = f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)$$

$$= \frac{1}{16} - \frac{1}{4} = -\frac{3}{16}.$$

$\therefore$ The tangent line passes through $\left(\frac{1}{4}, -\frac{3}{16}\right)$ and it has slope $-\frac{1}{2}$.

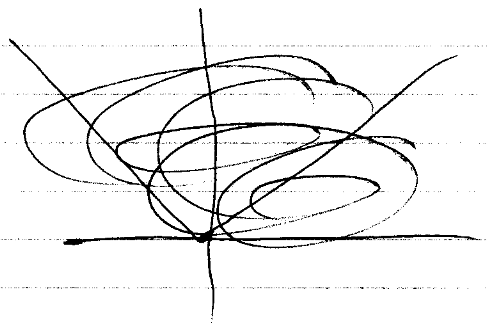
$\therefore$ Equation of the tangent line is,

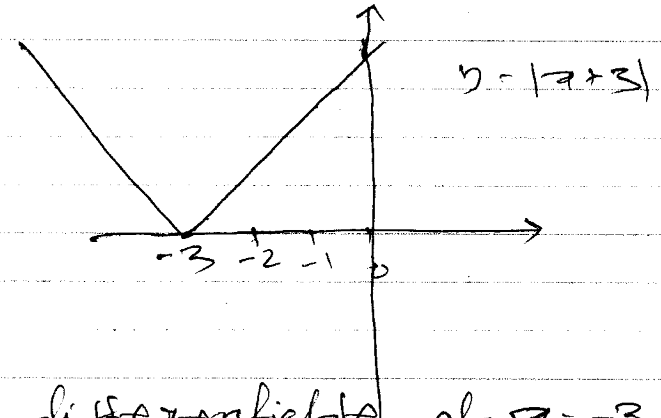
$$y - \left(-\frac{3}{16}\right) = -\frac{1}{2}\left(x - \frac{1}{4}\right)$$

i.e. $y + \frac{3}{16} = -\frac{1}{2}\left(x - \frac{1}{4}\right)$

i.e. $16y + 3 = -8x + 2$ i.e. $16y + 8x + 1 = 0$.

49) The function has a node at $x = -3$

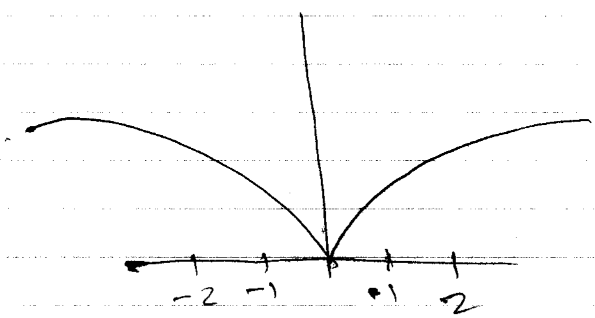




$\therefore$ The function is not differentiable at $x = -3$.

The function is differentiable everywhere else.

52) $y = x^{2/5}$

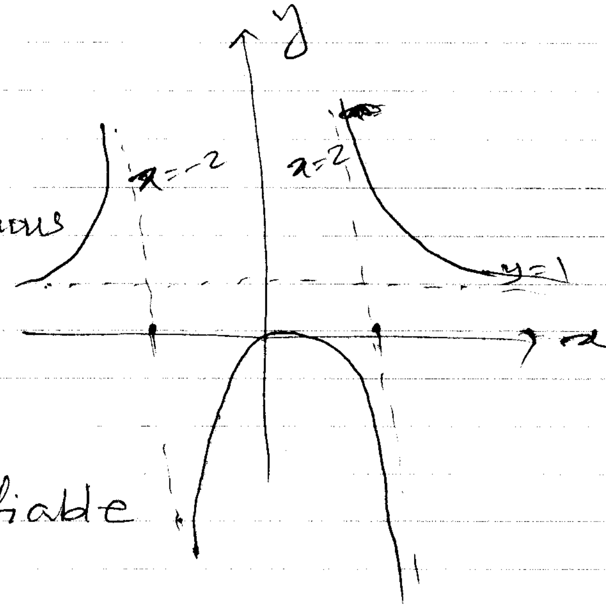


The function has a cusp at $x = 0$.

$\therefore$ It is not differentiable at $x = 0$.

It is differentiable everywhere else.

54) $y = \frac{x^2}{x^2-4}$



The function is discontinuous at $x = \pm 2$.

$\therefore$ It is not differentiable at $x = \pm 2$.

The function is differentiable everywhere else.