

## Section 1.6

[5]  $f(x) = \frac{1}{4+x^2}$  is a rational function. Therefore this function is continuous at each point in its domain. The function is defined when  $4+x^2 \neq 0$ . But notice that  $4+x^2 \neq 0$  for all real numbers. Therefore the domain of the function is the set of all real numbers. Consequently, the function is continuous everywhere.

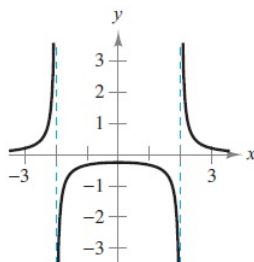
[8]  $f(x) = \frac{x+4}{x^2-6x+5}$  is a rational function. If  $x$  is in the domain of  $f$ , then we must have

$$\begin{aligned} x^2 - 6x + 5 &\neq 0 \\ \text{i.e., } (x-2)(x-3) &\neq 0 \\ \text{i.e., } x &\neq 2, x \neq 3. \end{aligned}$$

Hence the domain of the function is  $\{x \neq 2, x \neq 3\}$ . The function  $f$  is continuous at all real points except  $x = 2$  and  $x = 3$  i.e., the function is not continuous on the entire real line.

[10]  $g(x) = \frac{x^2-9x+20}{x^2-16}$  is a rational function. The domain of the function is all real except  $x^2 = 16$  i.e., all real except  $x = \pm 4$ . Hence the function is discontinuous at  $x = \pm 4$ . Consequently, the function is not continuous on the entire real line.

[12]  $f(x) = \frac{1}{x^2-4}$  is a rational function. The domain is all real except  $x = \pm 2$  i.e.,  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ . Therefore the function is continuous on  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ .



The above answer is also clear from the graph.

[13] From the following graph it is clear that the function is continuous on the interval  $(-\infty, -1) \cup (-1, \infty)$ .

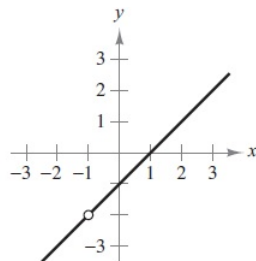


Figure 1: Problem 13

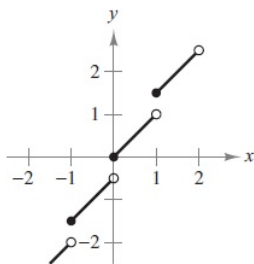


Figure 2: Problem 24

[24] From the above graph it is clear that the function is discontinuous only at integers. The function is continuous between two consecutive integers. Therefore the function is continuous in the intervals  $(m, m + 1)$ , where  $m$  is an integer.

[30] First of all, notice that the function is not defined for  $x = 4$ . We can rewrite the function as

$$f(x) = \frac{|4-x|}{4-x} = \begin{cases} \frac{(4-x)}{4-x}, & \text{if } 4-x > 0 \\ \frac{-(4-x)}{4-x}, & \text{if } 4-x < 0 \end{cases} = \begin{cases} 1, & \text{if } x < 4 \\ -1, & \text{if } x > 4. \end{cases}$$

Notice that the function is a constant on the either side of  $x = 4$ . Therefore the function is continuous on the either side of  $x = 4$ . The possible point of discontinuity is  $x = 4$ . At the beginning, we have already noticed that the function is not defined for  $x = 4$ . Therefore the function can not be continuous at  $x = 4$ . Therefore the function is continuous on  $(-\infty, 4) \cup (4, \infty)$ .

[38] Notice that the function  $f(x) = \frac{x}{x^2-4x+3}$  is defined for  $x^2 - 4x + 3 \neq 0$  i.e.,  $x \neq 1$

and  $x \neq 3$ . It is a rational function. Therefore it is continuous everywhere except at  $x = 1, 3$ . In particular it is continuous on  $[0, 4]$  except at  $x = 1, 3$  i.e., it is continuous on  $[0, 1) \cup (1, 3) \cup (3, 4]$ .

We also observe that  $\lim_{x \rightarrow 1} \frac{x}{x^2 - 4x + 3}$  and  $\lim_{x \rightarrow 3} \frac{x}{x^2 - 4x + 3}$  does not exist because in both cases we have  $\frac{\text{number}}{0}$  situation. Therefore  $x = 1, 3$  are non removable discontinuities.

[46] Given function

$$f(x) = \begin{cases} 2, & \text{if } x \leq -1 \\ ax + b, & \text{if } -1 < x < 3 \\ -2, & \text{if } x \geq 3. \end{cases}$$

Clearly the function is continuous on each of the intervals  $(-\infty, -1)$ ,  $(-1, 3)$  and  $(3, \infty)$ . The possible points of discontinuities are  $x = -1$  and  $x = 3$ .

x=-1: We see that  $f(-1) = 2$  and

$$\lim_{x \rightarrow -1^-} f(x) = 2, \quad \lim_{x \rightarrow -1^+} f(x) = -a + b.$$

The function  $f$  will be continuous at  $x = -1$  if

$$-a + b = 2. \tag{1}$$

x=3: We have  $f(3) = -2$  and

$$\lim_{x \rightarrow 3^-} f(x) = 3a + b, \quad \lim_{x \rightarrow 3^+} f(x) = -2.$$

The function  $f$  will be continuous at  $x = 3$  if

$$3a + b = -2. \tag{2}$$

Solving equations (1) and (2) we have  $a = -1$  and  $b = 1$ .

[56] From the graph it is clear that the function is continuous on  $(0, \infty)$ .

