Section 1.5

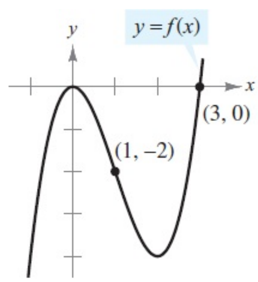
[1] Provided $f(x) = 5x + 4$.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	13.5	13.95	13.995	?	14.005	14.05	14.5

From the table

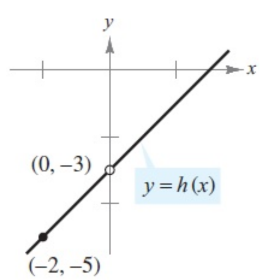
$$\lim_{x\to 2}(5x+4) = 14.$$

[10] From the graph it is clear that



$$\lim_{x\to 1} f(x) = -2$$
$$\lim_{x\to 3} f(x) = 0$$

[12] From the figure it is clear that



$$\lim_{x\to -2} h(x) = -5$$
$$\lim_{x\to 0} h(x) = -3.$$

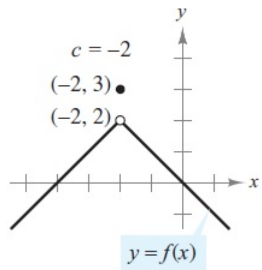
[17] It is easy to see from the graph that

$$\lim_{x\to 3^+} f(x) = 1$$
$$\lim_{x\to 3^-} f(x) = 1.$$

Therefore the limit exists and

$$\lim_{x\to 3} f(x) = 1.$$

[20] From the graph we see that



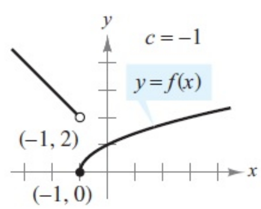
$$\lim_{x\to -2^+} f(x) = 2$$
$$\lim_{x\to -2^{-1}} f(x) = 2.$$

Therefore the limit exists and

$$\lim_{x\to -2} f(x) = 2.$$

Notice that the actual value of the function at -2 is 3 i.e., $f(-2) = 3$. But the limiting value is 2 i.e., $\lim_{x\to -2} f(x) = 2$. Interesting! we will talk about this in continuity chapter.

[22] From the graph we see that



$$\lim_{x\to -1^+} f(x) = 0$$
$$\lim_{x\to -1^-} f(x) = 2.$$

Since $\lim_{x\to -1^+} f(x) \neq \lim_{x\to -1^-} f(x)$, therefore $\lim_{x\to -1} f(x)$ does not exist.

[29]

$$\lim_{x\to 3} \sqrt{x+1} = \sqrt{4} = 2.$$

[41] Notice that this is a $\frac{0}{0}$ case.

$$\begin{aligned}\lim_{x\to -1}\frac{x^2-1}{x+1} &= \lim_{x\to -1}\frac{(x+1)(x-1)}{x+1}\\ &= \lim_{x\to -1}(x-1) = -2.\end{aligned}$$

[50] Notice that

$$|x-2| = \begin{cases} (x-2), & \text{if } x-2>0\\ -(x-2), & \text{if } x-2<0.\end{cases}$$

Therefore we have

$$\begin{aligned}\frac{|x-2|}{x-2} &= \begin{cases} \frac{(x-2)}{x-2}, & \text{if } x-2>0\\ \frac{-(x-2)}{x-2}, & \text{if } x-2<0\end{cases}\\ &= \begin{cases} 1, & \text{if } x>2\\ -1, & \text{if } x<2.\end{cases}\end{aligned}$$

So the limits are following

$$\lim_{x\to 2^+}\frac{|x-2|}{x-2} = 1$$
$$\lim_{x\to 2^-}\frac{|x-2|}{x-2} = -1.$$

Since $\lim_{x\to 2^+}\frac{|x-2|}{x-2} \neq \lim_{x\to 2^-}\frac{|x-2|}{x-2}$, therefore $\lim_{x\to 2}\frac{|x-2|}{x-2}$ does not exist.

[51] Given

$$f(x) = \begin{cases} \frac{1}{3}x-2, & \text{if } x\le 3\\ -2x+5, & \text{if } x>3.\end{cases}$$

It is easy to see that

$$\lim_{x\to 3^-} f(x) = \frac{3}{3} - 2 = 1 - 2 = -1$$
$$\lim_{x\to 3^+} f(x) = -(2\times 3)+5 = -1.$$

Therefore we have

$$\lim_{x\to 3} f(x) = -1.$$