Section 3.6

[1] Given function $f(x) = \frac{x^2+1}{x^2}$. Setting denominator to zero we get $x = 0$. The numerator $x^2 + 1$ is not zero at $x = 0$. So, $x = 0$ is a vertical asymptote.

To find horizontal asymptote we compute

$$\begin{aligned} \lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \frac{x^2+1}{x^2} \\ &= \lim_{x\to\infty} \left[1 + \frac{1}{x^2}\right] \\ &= 1. \end{aligned}$$

Therefore the horizontal asymptote is $y = 1$.

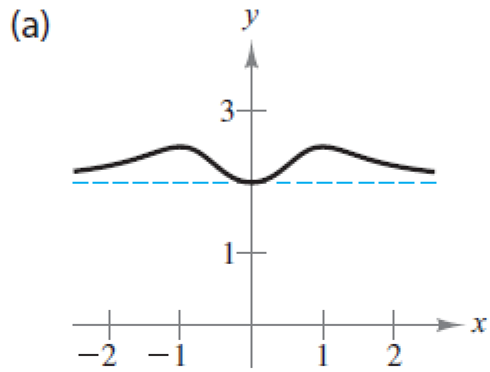
[7] Given function $f(x) = \frac{x^2-1}{2x^2-8}$. Setting denominator to zero we get $2x^2 - 8 = 0$ i.e., $x = \pm 2$. We also notice that $x^2 - 1$ is not zero at $x = \pm 2$. Therefore there are two vertical asymptotes, namely $x = 2$ and $x = -2$. To find the horizontal asymptote we compute

$$\begin{aligned} \lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \frac{x^2-1}{2x^2-8} \\ &= \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{2-\frac{8}{x^2}} \\ &= \frac{1-0}{2-0} \\ &= \frac{1}{2}. \end{aligned}$$

Therefore equation of the horizontal asymptote is $y = \frac{1}{2}$.

[12] Given function $f(x) = 2 + \frac{x^2}{x^4+1}$. Notice that there is no vertical asymptote. To find the horizontal asymptote we compute

$$\begin{aligned} \lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \left[2 + \frac{x^2}{x^4+1}\right] \\ &= 2 + \lim_{x\to\infty} \frac{\frac{1}{x^2}}{1+\frac{1}{x^4}} \\ &= 2 + \frac{0}{1+0} \\ &= 2. \end{aligned}$$

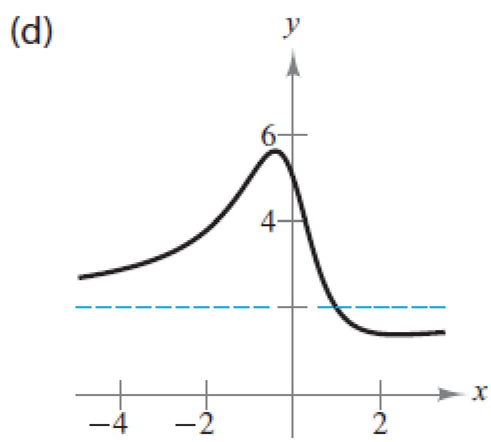


Equation of the horizontal asymptote is $y = 2$. We also notice that $f(0) = 2$. Graph of this function is graph (a).

[14] Given function $f(x) = \frac{2x^2-3x+5}{x^2+1}$. Notice that there is no solution of $x^2 + 1 = 0$. Therefore there is no vertical asymptote. To find the horizontal asymptote we compute

$$\begin{aligned}
\lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \frac{2x^2 - 3x + 5}{x^2 + 1} \\
&= \lim_{x\to\infty} \frac{2 - \frac{3}{x} + \frac{5}{x^2}}{1 + \frac{1}{x^2}} \\
&= \frac{2 - 0 + 0}{1 + 0} \\
&= 2.
\end{aligned}$$

Therefore the horizontal asymptote is $y = 2$. Notice that $f(0) = 5$. Therefore Graph of the



function is the graph (d).

[11] Given function $f(x) = x^3 - 6x^2 + 3x + 10$. To find the x intercepts we solve $f(x) = 0$

$$\begin{aligned} & x^3 - 6x^2 + 3x + 10 = 0 \\ i.e., \quad & x^3 + x^2 - 7x^2 - 7x + 10x + 10 = 0 \\ i.e., \quad & x^2(x+1) - 7x(x+1) + 10(x+1) = 0 \\ i.e., \quad & (x^2 - 7x + 10)(x+1) = 0 \\ i.e., \quad & (x-2)(x-5)(x+1) = 0 \\ i.e., \quad & x = -1, 2, 5. \end{aligned}$$

Now differentiating f with respect to x we obtain

$$f'(x) = 3x^2 - 12x + 3,$$

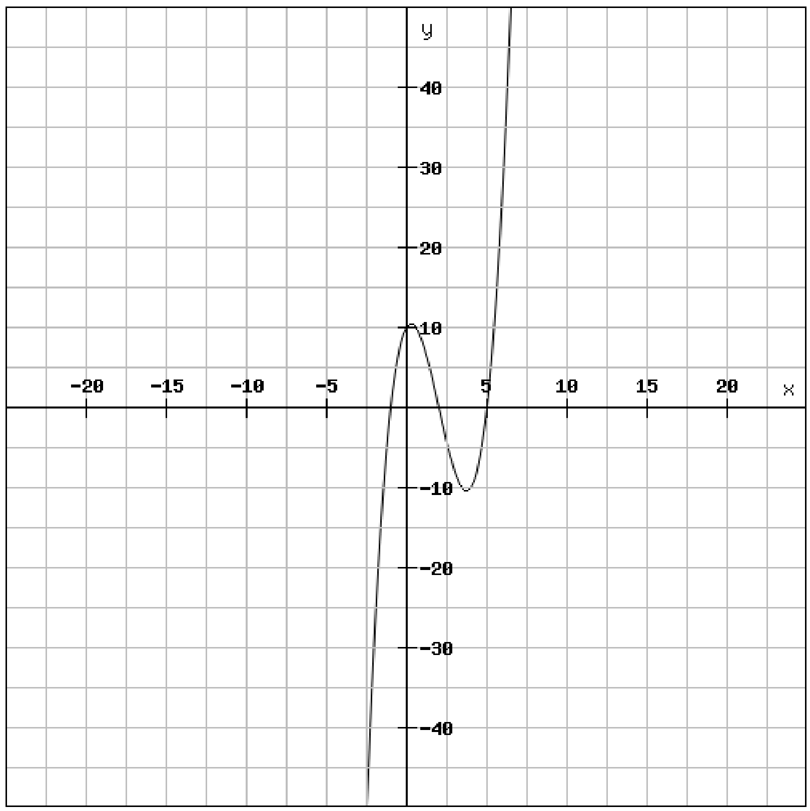
and

$$f''(x) = 6x - 12.$$

Solving $f'(x) = 0$ we get $x = 2 \pm \sqrt{3}$, and solving $f''(x) = 0$ we get $x = 2$. Key points are $x = -1, 2 - \sqrt{3}, 2, 2 + \sqrt{3}, 5$. We do the following test

	$f(x)$	$f'(x)$	$f''(x)$	**Characteristics of graph**
$-\infty < x < -1$		$+$	$-$	Increasing, concave downward
$x = -1$	0	$+$	$-$	Increasing, x intercept
$-1 < x < 2 - \sqrt{3}$		$+$	$-$	Increasing, concave downward
$x = 2 - \sqrt{3}$	$\sqrt{3}(3 + \sqrt{3})^2$	0	$-$	relative maximum
$2 - \sqrt{3} < x < 2$		$-$	$-$	Decreasing, concave downward
$x = 2$	0	$-$	0	Point of inflection
$2 < x < 2 + \sqrt{3}$		$-$	$+$	Decreasing, concave upward
$x = 2 + \sqrt{3}$	$-6\sqrt{3}$	0	$+$	Relative minimum
$2 + \sqrt{3} < x < 5$		$+$	$+$	Increasing, Concave upward
$x = 5$	0	$+$	$+$	Increasing, x intercept
$5 < x < \infty$		$+$	$+$	Increasing, concave upward

Finally, combining all the above information we have the following graph.


y
40
30
20
10
-20
-15
-10
-5
5
10
15
20
x
-10
-20
-30
-40